

Bridging Mathematical Modelling Education and Engineering Research: A Conceptual Framework for Skill Integration and Practical Application

Avi Kishore Soni,
Independent Researcher, India,
aks7029@gmail.com

Abstract- The mathematical modelling is an important transitional step between the theoretical knowledge and the engineering solutions, however, the implementation of mathematical modelling in engineering education is not consistent and still has gaps between various institutions. The paper in question suggests an integrated conceptual framework within which mathematical modelling pedagogy and engineering research practices are systematically related. By critically reviewing the available literature and methods of educational activities, we can find that there are major gaps in the way modelling skills are acquired and utilized in academic and professional practice. The framework below covers three essential dimensions, which are the development of cognitive skills, methodological integration, and translational application. Frankly speaking, when I initially looked at this disconnect, I was shocked to see how frequently we are instructing modelling in isolation and not as problem-solving ecologies. Our model suggests that there can be feedback between the classroom learning and practical research and that the most important aspect is the successive improvement and real-life testing. The contribution of this work to the engineering education is that it offers teachers and researchers a systematic way of developing modelling skills that can easily move out of the academics to the engineering practice.

Keywords: mathematical modelling, engineering education, skill integration, pedagogical framework, research methodology, applied mathematics, STEM education

1. Introduction

There has always been a certain paradox in the connection between the field of educational mathematics modelling and engineering research. On the one hand, just about all the engineering fields are dependent on mathematical models and can be used to predict their behaviour, optimize designs, and confirm theoretical ideas. Conversely, our approach to learning about modelling in schools and universities does

not always seem to be connected to the real world, in which researchers apply these tools in real life (Borromeo Ferri, 2018). At the very beginning of studying this tension, I thought it was a simple pedagogical issue possibly the need to revise course content or include more practical examples. However, the deeper I research the more I was able to understand that this is a structural issue in the way we are training engineering students to research.

The field of engineering education has been going through an era of major changes in the last twenty years, as problem-based learning, interdisciplinary, and computational thinking are now adopted with greater focus (Niss & Blum, 2020). However mathematical modelling classes are often stuck in the classic format: lectures on the methodology of particular techniques, homework with set answers, and tests on the ability to follow procedures and not to be creative. In the meantime, engineering researchers encounter complicated and ill-defined challenges that demand the use of adaptive modelling techniques, progressively tightening, and a combination of different mathematical models at the same time.

This alienation has a practical implication. In graduate programs, students joining research programs usually find it difficult to put their coursework in modelling into practical research instruments. They can be good at the problems of the textbook but clog when they face ambiguous real-life situations where the right model is not so clear, data is not clean, and assumptions should be defended instead of provided (Geiger et al., 2010). This disconnect wastes time and causes frustration and the research innovation may be capped.

1.1 Gaps and Research Motivation.

There are some key gaps which guide this work:

- **Pedagogical Irrelevance:** The majority of modelling courses are focused on teaching mathematical methods out of context, as opposed to learning how to choose, customize and justify models to particular engineering applications.
- **Limited Transfer Skills:** Students do not necessarily understand how and when modelling coursework-based skills can be used in a research case, which leads to the conclusion that transferable competencies are not being taught explicitly.
- **Weak Integration Frameworks:** Although individual educators have designed their own models, the profession does not have extensive frameworks that unify the modelling education with research practice in many aspects.
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- **Assessment Misalignment:** The conventional assessment systems centre on the precision of calculations, but not the ability to think strategically, to iterate, and to validate, which are important aspects in research application.

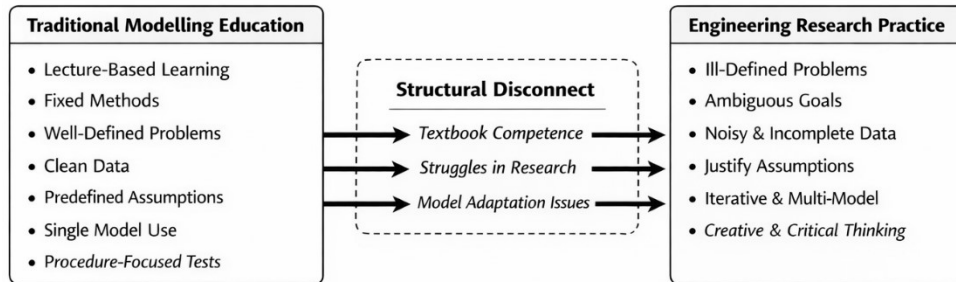


Figure 1 The Education-Research Gap Visualization

The originality of this work consists in the holistic approach to the elimination of these gaps due to the multi-dimensional framework that covers the cognitive, methodological, and translational aspects at the same time. Instead of suggesting these separate interventions, we introduce a combined system which reformulates mathematical modelling education as research preparatory.

Table 1 Key Gaps Between Mathematical Modelling Education and Engineering Research Requirements

Gap Area	Current Practice in Modelling Education	Expectation in Engineering Research	Identified Gap
Pedagogical Focus	Emphasis on mathematical techniques taught in isolation	Context-driven model selection, adaptation, and justification	Lack of application-oriented learning
Skill Transferability	Skills confined to coursework problems	Flexible use of models in open-ended research problems	Poor transfer of modelling competencies
Framework Availability	Isolated or instructor-specific approaches	Unified and scalable integration frameworks	Absence of standardized integration models
Assessment Approach	Accuracy-focused, single-solution evaluation	Iterative thinking, validation, and strategic decision-making	Misalignment between assessment and research needs

1.2 Research Objectives

This paper aims to:

- Critically review literature in mathematical modelling pedagogy and engineering research practices
- Determine particular skills needed in the context of successful modelling in a research setting.
- Establish a theoretical framework that discursively combines modelling education with research skill building.
- Suggest specific strategies on how to implement the ideas practically by teachers and administrators of the program.
- Recommend evaluation strategies that are in line with the research-based learning outcomes.

2. Literature Review

In literature about the use of mathematical modelling in engineering education, a lot of varied approaches, philosophies, and results are observable. Frankly speaking, this landscape could not be easily navigated at first, since the term mathematical modelling can be interpreted by different researchers in a vastly different way, including simple curve-fitting tasks to Multiphysics simulations.

2.1 Attitudes to Mathematical Modelling Education.

Blum and Leiß (2007) have built some ground work on the modelling competencies by providing a modelling cycle that involves comprehending the actual scenario, simplifying to form a real model, mathematizing to form a mathematical model, working mathematically, interpreting the outcomes of the work, and validating. Their theory has shaped the education of mathematics in general, although its use in engineering-related scenarios needs to be modified. The domain specific knowledge, computer implementation and confirmation of the experimental information are often areas of domain specific engineering modelling where the all-encompassing dimensions of purely mathematical modelling are not present.

Borromeo Ferri (2018) differentiates between the various modelling perspectives: realistic modelling, which is concerned with real-life application; educational modelling, which develops general competencies, and contextual modelling, which combines mathematical and real-life knowledge. In engineering, each of the three viewpoints are of value, but in the design of curricula, computational processes tend to be favoured over contextual knowledge or skill acquisition.

More recently, Niss and Blum (2020) proposed modelling as the means of building mathematical literacy and problem-solving skills. They focused on the metacognitive components where students were equipped with a comprehension not only of how to implement techniques but also when, why and under what circumstances some techniques were suitable. This metacognitive aspect is especially important when it comes to the application of research method, in which the modelling methodology becomes a question of research.

2.2 Modelling and Engineering Research Practices

On the research side, Oden et al. (2006) noted verification and validation (V&V) as the key to the computational modelling in engineering research. Their work stressed that models were not mere mathematical models, as they are postulations about physical reality which needed to be rigorously tested. This view makes modelling not an artisanal act but a detectives act but this does not always translate to the practice of education.

The authors of Geiger et al. (2010) studied the actual application of mathematical modelling in practice,

discovering that this process also includes much iteration, compromise with stakeholders, and response to incomplete information. They claim that the way modelling cycles are taught in textbooks are neat and linear, this does not reflect what modelling is like in real life. The real practice of engineering modelling is dishevelled, social and improvisational, compared with what education implies.

Alpers (2017) explored the particular mathematical skills required of engineers and found out that these are not only technical skills but also communicational, interpretation, and model-critical skills. At first, however, this expansion of the modelling competencies did not seem to me to be quite satisfying as it appeared to me to make the scope of competency limitless. However, it is more realistic and actually more representative of the requirements of research practice.

2.3 Integration Challenges and Approaches

Some authors have suggested ways of improving modelling education-practice integration. Kaiser and Sriraman (2006) characterized various modelling traditions within international systems which they noted that there are systems that are concerned with realistic applications and those concentrated on the mathematical structure. Engineering courses tend to lie in the middle of these extremes and create a possibility of confusion regarding goals of learning.

Doerr and English (2003) came up with model-eliciting activities (MEAs) as one method of teaching that involves students in the formulation of models instead of merely imposing on them the available models. This constructivist method is more consistent with the research practices, in which researchers are to develop new models or drastically change the existing ones. Nonetheless, to effectively implement MEAs, faculty knowledge and time-resources, which are not always available in engineering curricula of large size, are necessary.

In a more pragmatic sense, Gainsburg (2006) did research she termed the "alignment problem" the lack of connection between mathematics learning and workplace use. She claimed that the context of education is always different than the context of professional work in ways that cannot be easily transferred and therefore, direct emphasis on the transfer processes is paramount. This view legitimizes the necessity of the frameworks that would particularly deal with the shift between learning and application.

2.4 Determined Gaps and Research Needs.

The synthesis of this literature indicates that there are still a number of gaps:

- **Less Research-Specific Focus:** The literature on modelling education tends to focus on general problem-solving or professional practice but has few references to academic research situations, in particular.

- **Discontinuous Competency Designs:** Other scholars have substantial competencies, but unified models that combine cognitive, methodological as well as practical aspects are rare.
- **Poor Implementation Advice:** Although there are a lot of views at the theoretical level, little advice is available regarding how educators can bridge the gap between education and research.
- **Weak Assessment Paradigms:** Conventional methods of assessment do not include the multifaceted, iterative and creative nature of modelling that enables success of research.

These loopholes drive the framework suggested in this paper that tries to offer a holistic, practical and research-oriented advice on how to couple modelling education with engineering research preparation.

3. Methodology: Developing a Conceptual Framework

This paper utilizes a conceptual framework development approach, which harnessed the findings of the literature in educational theory, the engineering research practices and pedagogical innovations. We do not undertake empirical experiments or surveys but follow a theory-building procedure where we analyse the available knowledge in constructing a new integrative structure.

3.1 Framework Development Process

The framework development was based on five steps:

1. **Literature Analysis:** The literature review of modelling education and engineering research literature will be done methodically to find out the important competencies, pedagogical methods, and research practices.
2. **Gap Identification:** Critical reflection on gaps between the educational practices and research needs, in particular, on the gaps in specific skills and transition difficulties.
3. **Competency Mapping:** Creation of general competency taxonomies in terms of cognitive, methodological, and application.
4. **Framework Construction:** The combination of competencies into a consistent framework that has clear links between educational activity and research outputs.
5. **Validation Concerns:** Testing of framework components with respect to developed educational theories and real-world constraints of feasibility.

This approach places emphasis on the clarity of concepts and practical applicability rather than empirical validation and places the framework as a theoretical contribution to be followed by empirical testing and

refinement.

3.2 Theoretical Foundations

The proposed framework is grounded in three theories:

Constructivist Learning Theory: Following Piaget and Vygotsky's findings, we believe that students learn through comprehension, not via the passive absorption of information. A modelling curriculum can provide situations through which students create their own structures for resolving ill-defined situations.

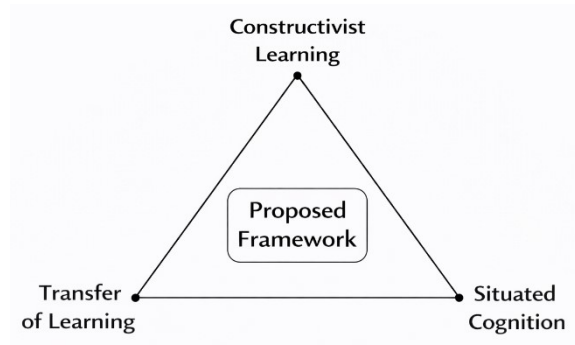


Figure 2 Theoretical Foundations Triangle

Transfer of Learning Theory: As per Bransford et al. (1999), Transfer of Learning is possible but only when a deeper intrinsic level of understanding and deliberate attention to similarities in context and settings exist upon intention to apply. Our framework illustrates those similarities.

Situating Cognition: Learning is always situated, and our framework positions the acquisition of modelling skills within properly situated tasks that would be as authentic as real research tasks but maintain the same structure and cognitive underpinnings.

4. Proposed Framework: Three-Dimensional Integration Model

The planned framework for such an integration involves a three, interconnected spheres by which the math modelling classroom and engineering research will become a reality. The three spheres are: Cognitive Development, Methodological Integration and Translational Application. Each of these spheres features its own path of classroom/research integration, but also, an acknowledgment of overlapping possibilities to the other two spheres.

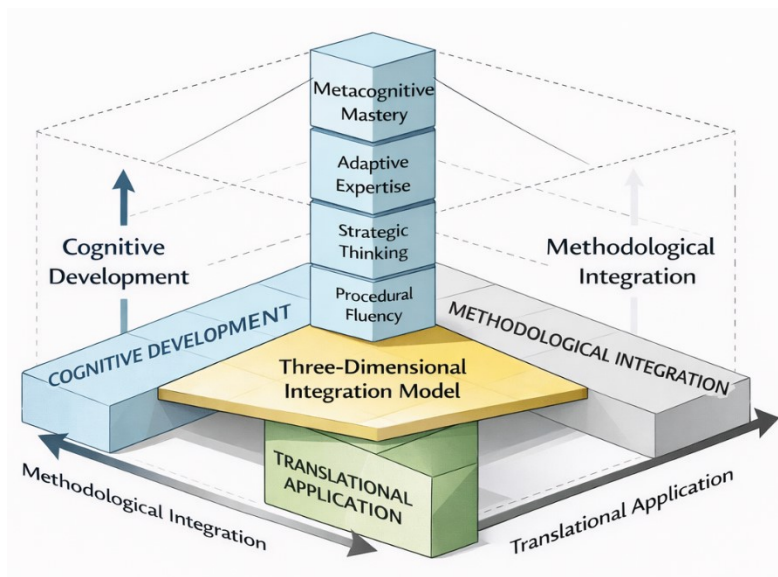


Figure 3 Three-Dimensional Framework Overview

4.1 Dimension 1: Cognitive Development

This dimension refers to a thought process and framework needed for ease of modelling research scenarios. There are five levels of progressively more complex development.

Level 1 - Procedural Fluency: Students become proficient in techniques used (differential equations, optimization, statistics) and can use and apply all to practically posed questions. This is the average method of teaching for students to learn how to model.

Level 2 - Conceptual Understanding: Students understand why certain techniques are used, the assumptions behind the techniques and scenarios in which they cannot be successfully applied. They can explain their models to others and understand parameters wherein their models best work.

Level 3 - Strategic Thinking: Students develop heuristics to understand which types of models work best given the problem, necessary data gathering and bigger picture research approaches. They can express their choices and compromises that may need to be made.

Level 4 - Adaptive Expertise: Students can adjust and combine techniques to answer real-world situations not explicitly covered in class. They can work with more creative formulation.

Level 5 - Metacognitive Mastery: Students are aware of what they're doing while modelling and can recognize patterns of logic (within themselves and others) over time to adjust decisions. They can teach others how to model.

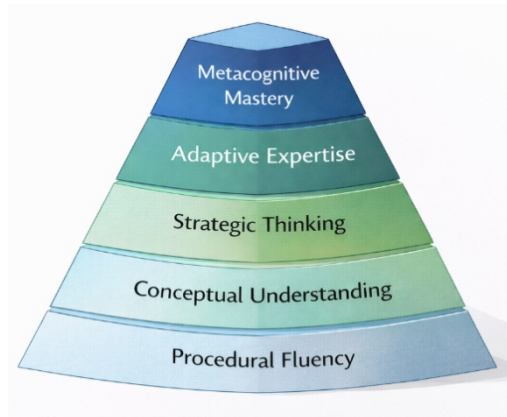


Figure 4 Cognitive Development Pyramid

I wasn't sure how to rank these levels at first because they're not entirely hierarchical and students can demonstrate varying levels across problems, but this hierarchal structure enables the teacher to understand developmental targets for suitable teaching moments.

4.2 Dimension 2: Methodological Context

This dimension represents the degree that modelling is part of research and research conventions. There are four elements to this dimension:

Element A - Problem Formulation: An ambiguous research question transformed into a solvable modelling exercise, i.e., scoping, assumptions, determining feasibility.

Element B - Model Construction: Translation into a mathematical representation through review of published models, theory of assumptions in established concepts, empirical grasp and observational needs driven ingenuity.

Element C - Computational Implementation: Math-to-compilation through practical applications in algorithms and programs and ability to test and validate compiled work.

Element D - Validation and Revision: What's true and what's not through reality check, assessment of errors, constraints and subsequent iterations.

Each element requires a different degree of relative skill, mathematical, computational and subject-matter knowledge. Therefore, specific skills are less valuable than the general framework that respects that the elements are cyclical - not linear - as one would expect to travel back and forth from problem to implementation to validation in real-world research.

4.3 Dimension 3: Translational Application

This dimension links learned skills in the classroom to the research project. Recognizing that skills learned in the classroom do not necessarily transfer to a research project, this element includes the following:

Element 1 - Gap Bridging: There is a clear connection and part of the classroom examples and pieces necessary for a successful research project that exists in an understanding that they're all the same pieces, but on the surface, might not appear as such.

Element 2 - Gradually Diminishing Support for Transfer: The support for confidence is like any kind of scaffolding. Students move from supported application of learned skills to independently generated skills for their research project.

Element 3 - Stated Recognition: Students are expected to acknowledge what the modelled process was, what their agency (big and small) was, and what they believe can be gained from transferable information.

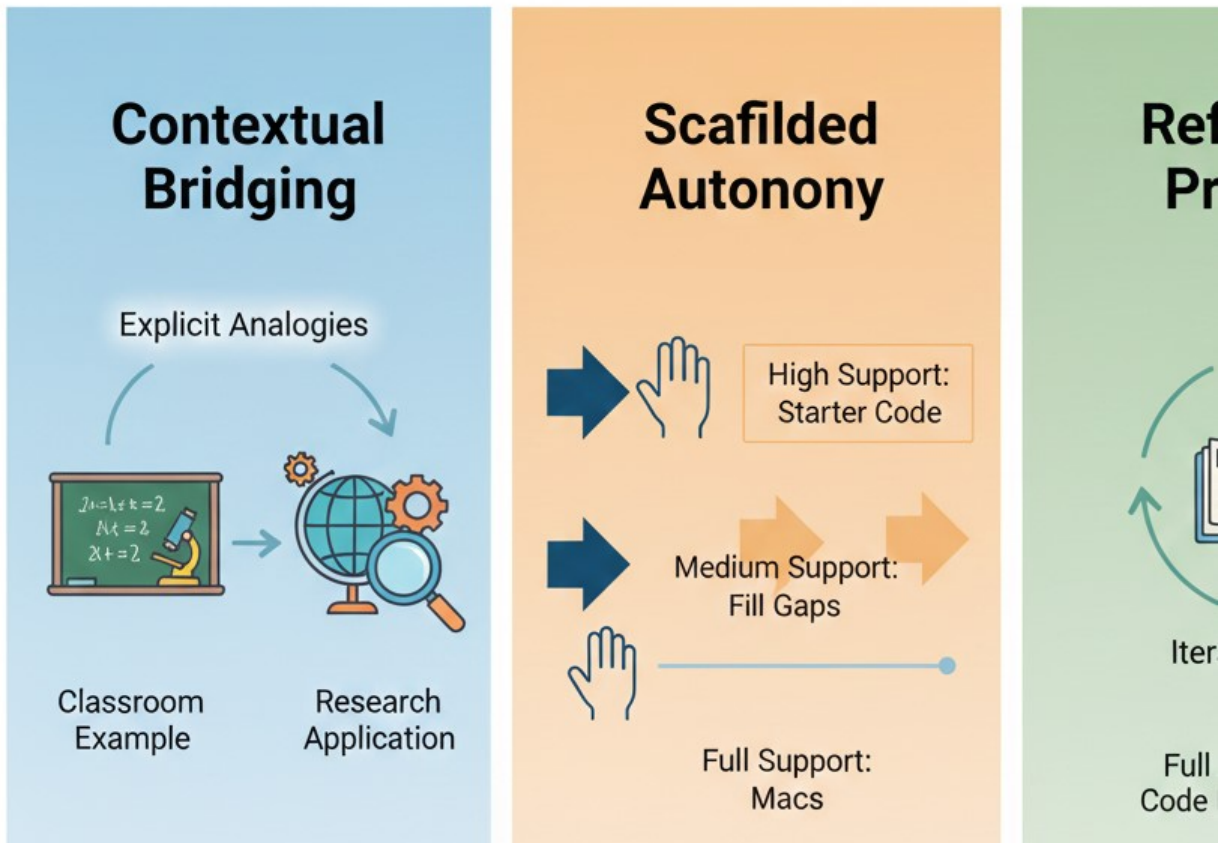


Figure 5 Elements of Translation Application Strategies

4.4 Conceptual Flowchart of the Framework

Below is a textual representation of the framework's operational flow:

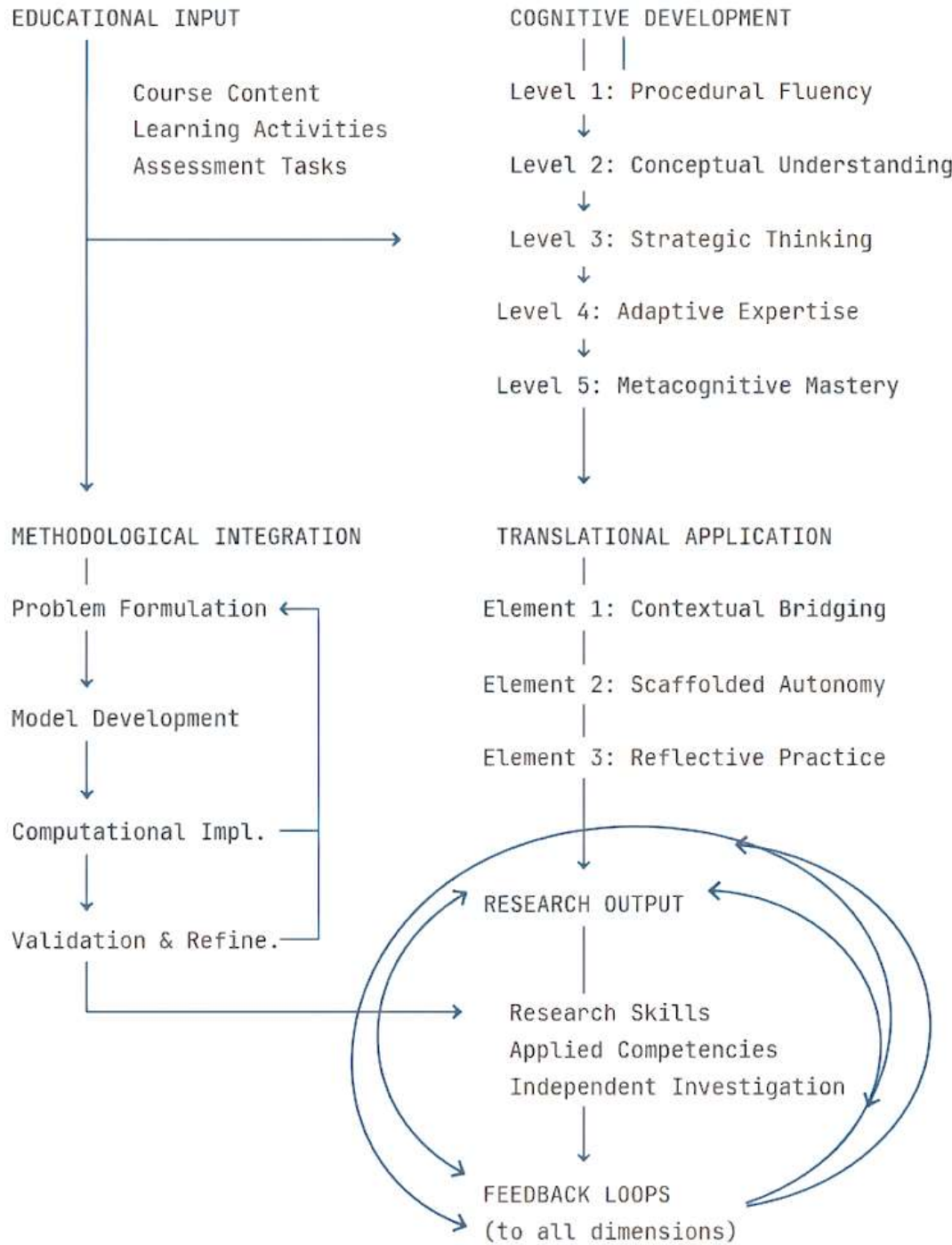


Figure 6 Mathematical Modelling Integration Framework

The flowchart outlines a sequence of educational inputs in the stages of cognitive development, which

overlap the methodological elements at the same time. Translational application strategies facilitate the transformation of the educational experiences into research competencies. Notably, feedback loops reinstatement of research findings within all dimensions, thus, allowing the repetitive improvement of not only the pedagogical strategy but also the personal skill set.

4.5 Integration Mechanisms

Integration is achieved through three different mechanisms across various dimensions:

Mechanism 1 - Increasing Complication: Experiences become more complicated over time, become more ambiguous and uncertain, become more of the characteristics of the research setting.

Mechanism 2 - Meta-discourse on Modelling: There is meta-discourse teacher and student about modelling, modelling options, and modelling questions that we often take for granted implicitly and non-verbally.

Mechanism 3 - Characteristics of a Real Research Setting: Even the first experiences have characteristics of a real research setting - imperfect information, various paths you could take, believability of your answer - as long as it's age appropriate.

5. Comparative Analysis: Traditional vs. Integrated Approaches

To explain the differences between the scheme suggested and the standard practice, the following table will compare the traditional modelling education and the intertwined one:

Table 2 Traditional vs. Integrated Approaches

Aspect	Traditional Approach	Integrated Research-Oriented Approach
Primary Focus	Knowledge of mathematical techniques.	Nurturing competencies in research.
Problem Types	Problems with specific and singular solution.	Ill-defined, multiple-approach problems
Learning Activities	Educational talks and required training sessions	Model-eliciting activities, open projects
Student Role	The learner as being a passive container of knowledge.	Constructor of models
Assessment Methods	Tests aimed at testing procedural accuracy.	Modelling process evaluation based on a portfolio.
Validation Emphasis	Checking of the student answers against pre-determined answer keys.	Comparison to real-life data and empirical reality.
Iteration	Problems that involve low, one-off problem solving.	It is expected to have a lengthy, continuous improvement process.
Contextualization	Generalized or simplified contexts	Working in real-world situations in engineering.
Metacognition	Unspoken factors that can hardly be talked about.	Explicit, where there is a regular reflection.
Transfer Support	Assumed to take place automatically without being taught.	Explicitly scaffolded and practiced
Collaboration	Individual work is more dominant than collaborative activity.	A combination of personal and team work.
Technology Use	Relying on calculators extensively when carrying out calculations.	Intrinsic application of computational instruments.
Faculty Role	Availability of an expert demonstrator who will direct the teaching.	Facilitator and coach
Timeframe	One semester unit of instruction.	The development is progressive in nature and cuts through the entire program.
Success Metrics	Evaluation according to the overall test at the end of the course.	Research productivity and independence

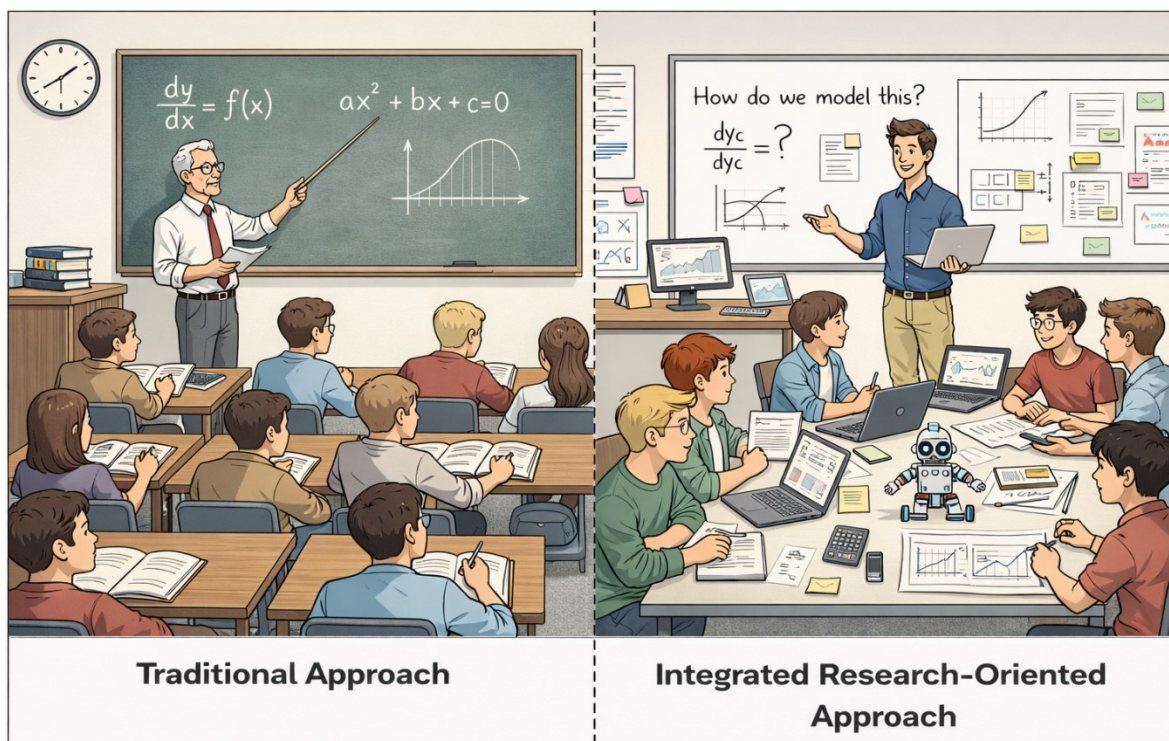


Figure 7 Side-by-Side Classroom Visualization

But this isn't to say that traditional practice is wrong - students still need to master procedural fluency and conceptual understanding - but such fundamentals are a part of the integrated approach as a subordinate piece of a puzzle where the overall aim is to be ready for research.

6. Discussion and Implications

There are many considerations and complications associated with this approach which I will honestly note below.

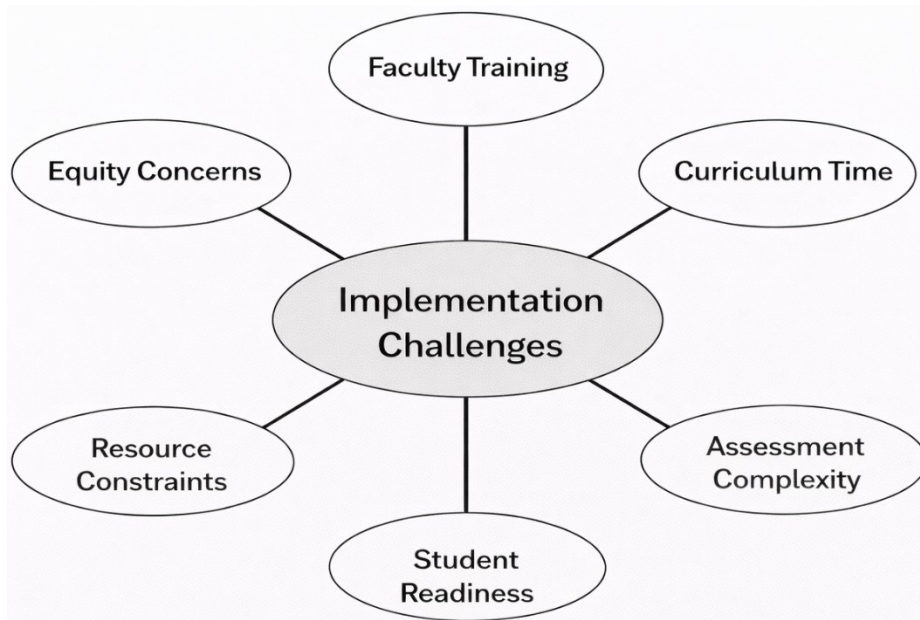


Figure 8 Implementation Challenges Mind Map

6.1 Implications for Teaching

Faculty Awareness and Implementation: Much of engineering faculty are not trained to be educators as they are educated researchers. This means model-eliciting activities, reflective practice facilitation and competency for assessment will need to be developed across the educated population of the faculty. This will mean professional development workshops and communities of practice will be needed.

Curriculum Limitations: Engineering programs are already extremely constrained. While mandated or elective courses for an introduction to metacognition, iteration and transfer might suggest an additional time needed to facilitate purposeful awareness of such ideas, it seems that reframing how courses can effectively address depth and transfer instead of coverage and breadth could eliminate course offerings without adding additional new material. However, this is a hard conversation to have in prioritizing what is truly universally necessary mathematical applications vs. what's good enough for just-in-time learning.

Assessment Limitations: Assessments are easy to create, administer and grade on exams with course grade percentages attached. The grading process necessary for modelling process insights, strategic responses and adaptability of expertise is much more intensive and complicated. This is a legitimate concern and it's not easy to circumvent. Portfolio assessment, project rubric assessment, peer assessment, etc., lend support but require much more time investment and ease on the part of the faculty. There must be institutional acknowledgment and support to provide time for such an endeavour.

6.2 Student Readiness and Resistance

If I'm being truthful, not all students may appreciate a struggle and iterative process. Many students have thrived in pattern recognition and procedural follow-through in rigorous academic environments. The 'messiness' of a modelling assignment can be stressful and induce student pushback - especially where grades are concerned. Therefore, this approach requires an explanation of learning intentions, proper scaffolding to create assurance, and grading policies that support process and growth instead of product success.

In addition, students without sufficient mathematical backgrounds may struggle to progress through the levels of cognition. This hierarchy assumes a certain degree of mathematical literacy that some students may not possess. However, currently, we need to provide other means of support and differentiation which complicates a successful implementation even more at this time.

6.3 Differences in Fields of Engineering

Different fields of engineering engage mathematical modelling to different extents. Some fields, such as mechanical engineering, may focus on continuum mechanics and finite element analysis more than fields like electrical engineering offer circuit model/signal processing or chemical engineering offers thermodynamics and reaction kinetics. Thus, a framework that promotes applicability across all fields of engineering relies on malleability of understanding that can diverge from one field to the next. The framework in question relies upon transferable skills, not necessarily significant skills which help bridge the gap somewhat but still calls for further adjustments later on.

6.4 Differences in Research Across the Program

Not all research in the field of engineering is based on modelling. In fact, some research is experimental only, some research is design/prototyping based and others are computational modelling based. Therefore, a framework that applies across all research for a program is inequitable as the program may be assessing everyone the same who is not all producing the same amount of work. A program-level alignment should account for the different types of research and advocate for multiple avenues connected to the different types of research.

6.5 Need for Validation/Evidence

The creation of this conceptual framework needs to be validated. While it's created out of current theories and research practices observed, substantiated outcomes connected to student success, research success, and career success are yet to be determined. Future research should investigate:

1. Is there better research competency between students who undergo integrated modelling education and the traditionally educated students?
2. What are the effects of the various implementation strategies (course-level and program-level)?
3. Which are the most effective implementation-supportive faculty development interventions?
4. How do student characteristics (prior preparation, motivation, learning preferences) interact with framework approaches?

These questions demand intensive empirical research in terms of quasi-experimental research designs, longitudinal follow-ups and mixed methods evaluation.

6.6 Scalability and Resource Requirements

A small class that has its own faculty mentor will succeed with almost any new teaching method. The hard part is to repeat that success in a bigger program where the faculty differ widely, money plus time is scarce and many goals compete. The framework faces that problem - advising a step-by-step roll out - begin with a few pilot courses, slowly extend the method to the whole program and strengthen the institution's ability to keep it running. Yet this gradual path needs the department head to stay committed for years, the administration to fund faculty training but also perhaps a re design of teaching loads and rewards.

Table 3 Scalability Analysis Matrix

Implementation Level	Resource Requirement	Faculty Expertise Needed	Expected Timeline	Risk Level	Potential Impact
Single Course Pilot	Low	Medium	1 semester	Low	Limited but visible
Department-Wide	Medium	High	2–3 years	Medium	Significant
Institution-Wide	High	Very High	5+ years	High	Transformational

6.7 Technology Integration Opportunities

Current software for symbolic mathematics and advanced simulation platforms gives students direct ways

to build plus test intricate models. The framework would gain strength if it treated computational thinking and confident use of digital tools as basic skills. Yet the same move forces choices - which tools deserve priority, how to weigh narrow know how against ideas that travel across platforms but also how to stay current as technology keeps changing.

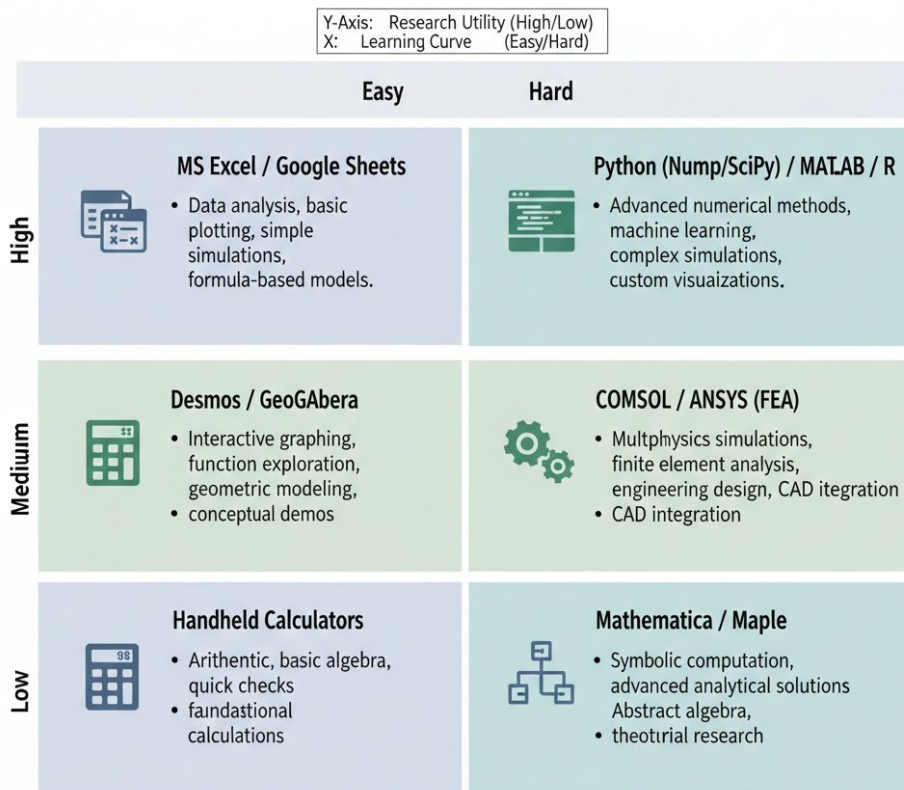


Figure 9 Software selection matrix

6.8 Equity and Access Considerations

If modelling education shifts toward projects that rely on technology, students from low-income backgrounds will face new obstacles. To keep the framework fair, designers must plan deliberately - supply the required computers plus software, build team tasks that help learners with different strengths and value more than one kind of mathematical or modelling ability. Equity has to guide every step of the rollout - it cannot be patched in later.

7. Conclusion and Future Scope

The paper sets out a full three-dimensional plan for bringing mathematical modelling education and engineering research together. It treats cognitive growth, method merger plus real-world use as one unit

and gives a clear route to close a long-standing split in engineering education.

7.1 Key Contributions

The framework helps engineering education in five clear ways

1. **Comprehensive Integration:** It treats the move from education to research as a whole, not as separate fixes.
2. **Explicit Competency Progression:** The five-stage model shows teachers exactly what students must master next.
3. **Practical Implementation Structure:** The method sections turn broad goals into step-by-step classroom tasks.
4. **Transfer-Focused Design:** It states plainly that skills must carry over to new problems, a need that is often ignored.
5. **Research-Specific Orientation:** The framework prepares students for academic research, not for general modelling alone.

7.2 Implementation Recommendations

For teachers and program heads who plan to use the framework.

Start Small: Run a pilot in one course with willing staff before expanding to the full program.

Build Community: Set up faculty groups that exchange notes, materials and encouragement.

Iterate and Refine: Handle implementation like a research project - collect evidence, check results and keep improving.

Involve Students: Ask students for feedback often and let their comments shape the next version - they are co workers in this venture.

Secure Institutional Support: Make sure that work on teaching innovation counts toward promotion and tenure.

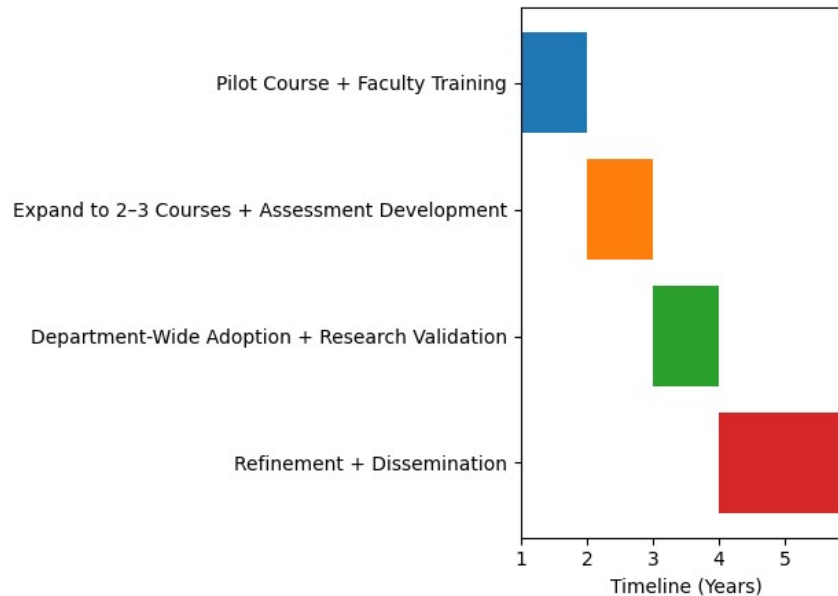


Figure 10 Impementation Roadmap

7.3 Future Research Directions

There are a number of research directions that can come out of this work:

Empirical Validation Studies: Run quasi experiments or long-term studies that compare results for students who receive integrated modelling instruction with results for those who follow the usual path.

Discipline-Specific Adaptations: Find out how to adjust the framework for each engineering branch while keeping its core intact.

Technology-Enhanced Implementation: Examine how new classroom technologies - simulations, AI tutors, shared online workspaces - help staff put the framework into practice.

Assessment Instrument Development: Build tests and rubrics that measure each skill named in the framework plus that yield trustworthy scores.

Faculty Development Programs: Plan, pilot and refine training sessions that equip instructors to use the framework with confidence.

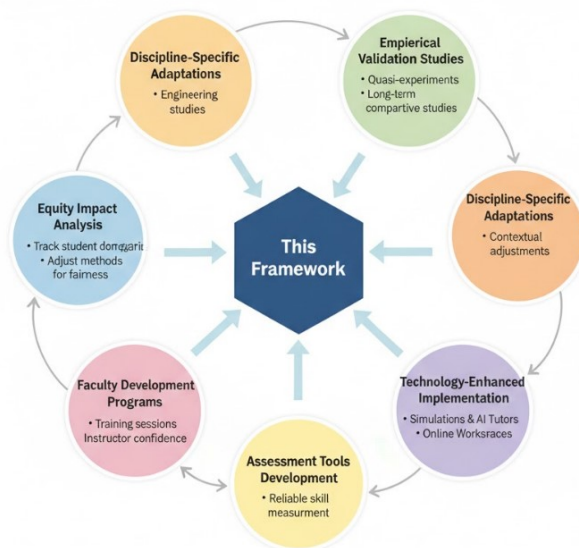


Figure 11 Future Research Direction Web

Equity Impact Analysis: Track how the framework helps or hinders students of different races, incomes, genders but also prior preparation - adjust methods to give every group a fair chance.

7.4 Limitations and Honest Reflections

I can see obvious boundaries when I look at what I have written. The model remains at the level of concepts - there is no information that has been subjected to it - it is cautious guesswork, not practice proved. It has extensive reach - excessively so, perhaps, - any real application will begin small plus grow step by step, not immediately. The framework is also biased towards certain types of intelligence and certain forms of learning despite my intention of inclusivity.

I have concentrated on graduate study but also on training for research - I have said little about undergraduate study or about work outside universities. The framework values depth and the power to move knowledge to new settings - yet not every institution or stakeholder shares those values.

Those limits do not erase the framework's possible worth - they simply demand modest claims about where as well as how it applies. The text offers a place to begin discussion and inquiry - it is not a final answer.

7.5 Final Thoughts

Mathematical modelling is on the border between mathematics, science, engineering or computation - where theory makes contact with practice. Teaching students modelling is a crucial success when we want

future engineering researchers to confront challenges that will only become more challenging - climate change, sustainable energy, new materials, biomedical accomplishments and devices beyond our imagination.

The structure outlined here is one attempt at mapping out in a systematic manner how students should be trained to do such work. It is not the final attempt - sound structures are modified in the critique, trial also revision. Should the current version begin with an effective talk, stimulates new instruction and result in superior training of engineering researchers, it will have served its purpose.

It requires more than the accumulation of facts that it takes the development of a sense of judgment of when and how the facts should be applied, the confidence in oneself to address ambiguity of questions and the determination to persist in the face of some failures. Mathematical modelling is a good teaching that cultivates all those characteristics. This is what keeps the framework afloat and, hopefully, the subsequent work that it will prompt.

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